The SI unit kilogram: the new definition and its realization on the basis of fundamental constants

At present, the International Prototype of the Kilogram, IPK, a 125 years old small cylinder of platinum-iridium, still defines the SI unit of mass. For 2018 it is proposed to redefine four of the seven SI units, in fact on the basis of fundamental constants. This is e.g. already done with the length unit meter which is referred to \( c \), the velocity of light. For the kilogram the *Avogadro experiment* provides an opportunity to link the kilogram to the atomic mass constant \( m_u \) by counting atoms in a given amount of mass – here a kilogram of a \(^{28}\text{Si} \) single crystal. And Avogadro’s number which is the number of entities in a mole would be the related fundamental constant. But how to “count” \( 10^{25} \) atoms as the age of the universe is only \( 10^{17} \) seconds? The solution is a crystal – a very good crystal of best purity, highest quality and perfection of crystalline order. With this it is possible to calculate the number of atoms if we only know the distance of the atoms in the crystal and if we know the macroscopic dimension of an artifact of this crystal. For the determination of Avogadro’s constant, a sphere made from a silicon crystal was chosen. Silicon crystals, which occur face-centered cubic, in high perfection are available since the early seventies due to semiconductor industries and the form of a sphere was selected as the obvious form of a cube failed because of the stability of its edges.

In a first step it is necessary to measure the Avogadro constant, \( N_A \), with best uncertainty so that, after fixing this value, in future the mass of an artifact will be determined through the fixed value of \( N_A \).

For the measurement of the Avogadro constant four quantities are to be determined:

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N_A = \frac{M_{\text{Si}} \cdot V_{\text{sphere}}}{n \cdot V_{\text{uc}} \cdot m_{\text{sphere}}}, \text{ with } n = 8 \text{ the number of atoms per cubic unit cell}
\]

Herein the quotient of macroscopic volume \( V_{\text{sphere}} \) and the volume of the unit cell \( V_{\text{uc}} \) of silicon atoms gives the number of atoms of that sphere, and molar mass \( M_{\text{Si}} \) and mass of the sphere \( m_{\text{sphere}} \) taking into account the mass of the entity so that the number of atoms per mole is derived.

The measurements are divided into crystal measurements which determine parameters typical for the whole silicon crystal, here the molar mass and the volume of the unit cell, and the properties which are related to the artifact produced from the crystal, here mass and volume of a test sphere.
To determine the molar mass with a resolution of better than $10^{-7}$ it is necessary to use isotope enriched material. In natural resources silicon consists of 92.23% $^{28}$Si, 4.67% $^{29}$Si and 3.1% $^{30}$Si, so it was decided to strike up a cooperation with Russian institutes to obtain silicon better than 99.99% $^{28}$Si – with the drawback of a price of about 1 Mio € per 1 kg sphere. For this material a new method, the isotope dilution mass spectrometry IDMS, could reduce the uncertainties for the molar mass determination to some few parts in $10^9$.

For the volume of the unit cell of silicon the lattice parameter is determined by combined X-ray and optical interferometry. Three thin probes of the crystal are arranged in a Laue interferometer where an X-ray beam is split and recombined from the first two plates so that the X-ray interference can be observed with the third plate, the analyzer. For this the analyzer is moved and its movement measured with an optical interferometer. The uncertainty for the unit cell volume reached $7 \times 10^{-9}$.

From each crystal of about 5 kg two spheres can be produced. They are manufactured with outstanding perfection – they reach small deviations from roundness, attain extreme small roughness and show no subsurface damage of the crystal lattice. These spheres have to be measured for mass and volume. For the mass the sphere is compared to the national prototype of kilogram, a Pt-Ir cylinder. As volume, surface and material of sphere and cylinder is distinctly different, numerous measurements against sorption artifacts and transfer standards are necessary. For the volume of the sphere an optical interferometer is used. It consists in the main of two high performance objectives with spherical reference faces which spacing is determined. In a second step the sphere is inserted in this spherical spacing and the resulting gaps between sphere and the respective objective are measured. This interferometer resolves deviations from roundness in the sub-nm range and yields full topographies of the silicon spheres. As this interferometer is unique rare and interesting images of the high precision spheres of the Avogadro project will be presented.

![fig. 2](image) $^{28}$Si single crystal sphere in the sphere interferometer of PTB